SELF-MAGNETIC FIELD EFFECTS ON ELECTRON EMISSION IN PLANAR DIODES AS THE CRITICAL CURRENT IS APPROACHED*

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Abstract

The self-magnetic field associated with the current in a planar diode is shown to reduce electron emission below the Child-Langmuir current density. As the magnetic field increases, the diode current is limited to the critical current. Here, a 1-D analysis is carried out to calculate the suppressed current density in the presence of a transverse magnetic field. The emitted current density is found to decrease modestly until the magnetic field associated with the critical current is reached, at which point the emission shuts off abruptly. The 1-D analysis remains valid until orbit crossing occurs as the current approaches the critical current. The minimum diode length required to reach critical current is also derived.

I. INTRODUCTION

High power vacuum diodes are used to produce intense electron beams for many applications. When self-magnetic field effects are negligible and the cathode is a space-charge-limited (SCL) emitter, Child-Langmuir flow is obtained[1]. Here, the effect of the diode self-magnetic field on the locally emitted current density is investigated. Results presented here are applicable to cylindrical diodes with an anode-cathode gap D that is much less than the diode cathode radius R_c. In this case, the diode can be treated as planar and, when the magnetic field is negligible, electrons cross the diode with straight-line trajectories as illustrated in Fig. 1a. However, when the self-magnetic field associated with the diode current becomes large enough to significantly deflect the electron path as it crosses the diode gap as illustrated in Fig. 1b, the increased electron space charge density in the gap above the emission site reduces the local emission below the Child-Langmuir current density. At high magnetic field, the diode current is limited to the critical current I_{crit}, which is obtained when the electrons reach the anode at grazing incidence (or equivalently, the "electron gyroradius" equals the gap size)[2,3]. Critical current flow is illustrated in Fig. 1c.

A 1-D analysis is used to calculate the suppressed current density in the presence of a transverse magnetic field. The problem is similar to that of calculating the Hull current in a crossed field gap[4]. In treating electron flow in crossed field gaps, the Llewellyn approach is typically used[4], whereas, here, Poisson's equation is solved by direct integration. Also, in the case considered here, the magnetic field is produced by the diode current itself and increases with distance along the gap causing the curvature of the electron trajectories to increase with distance as well (see Fig. 1c).

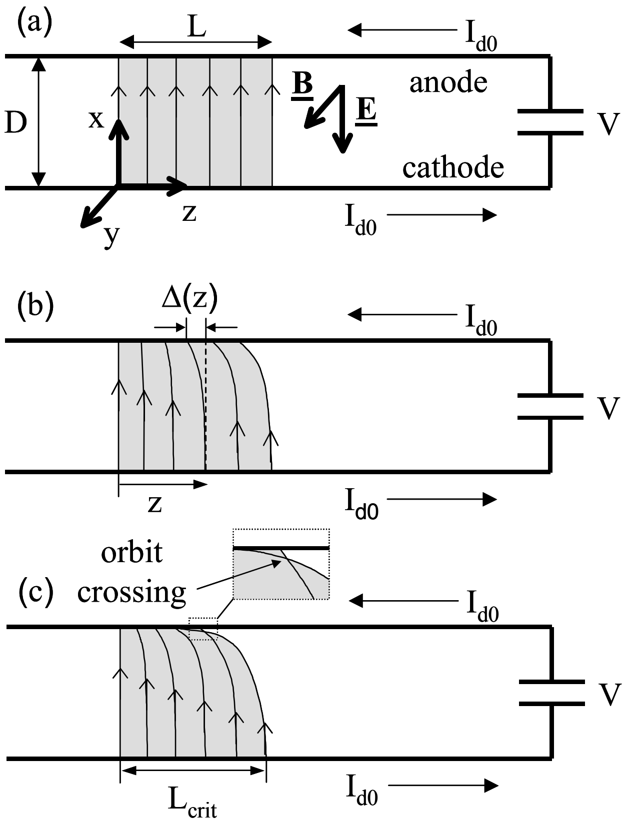

Figure 1. Schematic of diode electron flow for (a) $I_{d0} \ll I_{crit}$, (b) $I_{d0} < I_{crit}$, and (c) $I_{d0} = I_{crit}$.

This problem is of interest for determining the minimum length L_{crit} of a diode designed to run at I_{crit}. This situation arises when an electron beam pinch is desired to concentrate the electron-beam energy, while minimizing losses to ion current which are proportional to the diode length in cylindrical diodes[5] (or diode radius in pinched-beam diodes[6]). Although this analysis does not include ion current, the desired minimum diode length with ion current can still be reasonably estimated. For a nonrelativistic planar diode, it is well known that the SCL electron current density is enhanced by a factor of 1.86

* Work supported by ONR and DOE through SNL, LANL, and LLNL.
** JAYCOR, Inc, McLean, VA 22102.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
JUN 2001	**N/A**	**-**

4. TITLE AND SUBTITLE	
Self-Magnetic Field Effects On Electron Emission In Planar Diodes As The Critical Current Is Approached	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. IEEE International Pulsed Power Conference (19th). Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.

14. ABSTRACT

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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**SAR**	**4**	

when ion current flows[7], while the scaling with diode voltage and gap remains the same. Thus, it is assumed here that the minimum diode length for bipolar flow L_{crit}^{BP} is to first order simply $L_{crit}^{BP} = L_{crit}/1.86$.

II. SOLUTION WITH $B_0 \neq 0$

The planar diode geometry considered here is illustrated in Fig. 1. A voltage V is applied across the electrodes, and a uniform magnetic field is applied transverse to the electron flow in the y direction. The current I_{d0} is emitted over an axial length L as shown in Fig. 1a. The problem can be cast in terms of the electric potential $\Phi(x)$ and the magnetic vector potential $A_z(x) = -B_0x$, where $B_0 > 0$, $\Phi(0) = 0$, and $\Phi(D) = V$. For SCL flow in equilibrium, $E_x(0) = d\Phi/dx(0) = 0$. Because y and z are ignorable coordinates, their canonical momenta are conserved. An expression for $v_x(x)$ is obtained from conservation of energy. Thus,

$$v_x^2(x) = \frac{2e\Phi(x)}{m_e} - \omega_{ce}^2 x^2 \ , \qquad v_y(x) = 0 \ , \ \text{and} \ \ v_z(x) = -\omega_{ce} x \ , \tag{1}$$

where $\omega_{ce} = eB_0/m_ec$, e and m_e are the electron charge and mass, c is the speed of light, and $v_x(0) = v_y(0) = v_z(0) = 0$. Note that for $B_0 = 0$, $v_x(D) = v_0 = (2eV/m_e)^{1/2}$, which is the speed obtained by an electron in crossing the gap.

Because $v_x^2 \geq 0$, Eq. (1) shows that $I_{d0} \leq I_{crit}$ (or $\omega_{ce} \leq \omega_{ce}^{crit}$) where

$$I_{crit} = \left(\frac{m_e c^4}{2e}\right)^{1/2} V^{1/2} \frac{R}{D} \ , \qquad \omega_{ce}^{crit} = \left(\frac{2eV}{m_e}\right)^{1/2} / D = v_0/D \ , \tag{2}$$

and $B_0 \equiv 2I_{d0}/cR_c$ so that these planar results can be related to problems in cylindrical geometry with $R_c >> D$.

In equilibrium with a uniform applied magnetic field (so that $d/dz = 0$), the continuity equation shows that the current density $J_x(x) = J_0$ is constant and

$$n_e(x) = \frac{-J_0}{ev_x(x)} \ , \tag{3}$$

with $J_0 > 0$. In the next section where B_0 is allowed to vary in z along the diode length, it is assumed that the equations for v_x and n_e in Eqs. (1) and (3) still apply as long as $d/dx >> d/dz \sim 0$. Using the expression for n_e given in Eq. (3), Poisson's equation becomes

$$\frac{d^2\Phi}{dx^2} = \frac{4\pi J_0}{\sqrt{\dfrac{2e\Phi}{m_e} - \omega_{ce}^2 x^2}} \ . \tag{4}$$

Here, however, it is convenient to rewrite the equation as

$$\frac{d^2S}{dx^2} = \frac{8\pi e J_0}{m_e\sqrt{S}} - 2\omega_{ce}^2 \ , \tag{5}$$

where $S(x) = v_x^2(x)$ and $v_x^2(x)$ is given in Eq. (1). The first integral can be obtained by multiplying both sides of Eq. (5) by dS/dx and integrating. This yields

$$\frac{dS}{dx} = \pm\left(\frac{32\pi e J_0}{m_e}\sqrt{S} - 4\omega_{ce}^2 S\right)^{1/2} \ , \tag{6}$$

where the two boundary conditions, $S(0) = 0$ and $dS/dx(0) = 0$, have been used. The upper sign applies in the case when v_x is increasing, while the lower sign applies when v_x is decreasing. This occurs when $I_{d0} \to I_{crit}$ where the electron orbit has been significantly deflected by the magnetic field so that energy is being transferred into axial motion as the electron approaches the anode. The second integral can be obtained by direct integration, yielding for $dS/dx > 0$

$$x = -\left[\frac{8\pi e J_0 v_x(x)}{m_e\omega_{ce}^4} - \frac{v_x^2(x)}{\omega_{ce}^2}\right]^{1/2} + \frac{4\pi e J_0}{m_e\omega_{ce}^3}\left\{\mathrm{Sin}^{-1}\left[\frac{m_e\omega_{ce}^2 v_x(x)}{4\pi e J_0} - 1\right] + \frac{\pi}{2}\right\} \ , \tag{7}$$

where $v_x(x)$ is given in Eq. (1) and $S(0) = v_x^2(0) = 0$ has been used. This solution applies for $x < x^{(1)}$ where $x^{(1)} = 4\pi^2 eJ_0/m_e\omega_{ce}^3$ is defined as the position where v_x stops increasing (i.e., where $dS/dx = 0$). The value of $x^{(1)}$ depends on the magnetic field strength through $J_0(\omega_{ce})$ and ω_{ce} directly where $\omega_{ce} = \omega_{ce}^{(1)}$ when $x^{(1)} = D$. It can be shown[8] that $\omega_{ce}^{(1)} = \pi\,\omega_{ce}^{crit}/(\pi^2 + 2)^{1/2}$.

When $x^{(1)} > D$, Eq. (7) applies for all $0 \leq x \leq D$. When $x^{(1)} \leq D$, Eq. (7) applies for $0 \leq x \leq x^{(1)}$ and Eq. (6) must be solved again (using the lower sign) for $x^{(1)} \leq x \leq D$. That solution is then matched to the solution in Eq. (7) at $x = x^{(1)}$, yielding

$$x = +\left[\frac{8\pi e J_0 v_x(x)}{m_e\omega_{ce}^4} - \frac{v_x^2(x)}{\omega_{ce}^2}\right]^{1/2} - \frac{4\pi e J_0}{m_e\omega_{ce}^3}\left\{\mathrm{Sin}^{-1}\left[\frac{m_e\omega_{ce}^2 v_x(x)}{4\pi e J_0} - 1\right] - \frac{3\pi}{2}\right\} \ . \tag{8}$$

Assuming that $x^{(1)} > D$, the final boundary condition $\Phi(D) = V$ can be applied to Eq. (7) to find the eigenvalue

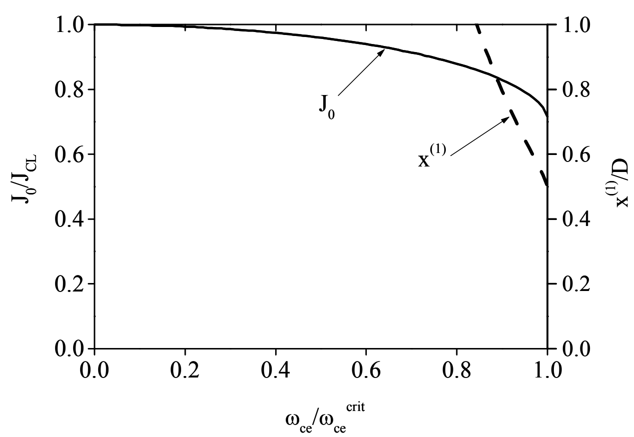

Figure 2. Plots of current density J_0/J_{CL} (solid) and $x^{(1)}/D$ (dash) as functions of $\omega_{ce}/\omega_{ce}^{crit}$.

$J_0(\omega_{ce})$. Using J_0, Eq. (7) then can be solved numerically for $\Phi(x)$. When $x^{(1)} < D$, Eq. (8) with $\Phi(D) = V$ is used to find $J_0(\omega_{ce})$, and then Eqs. (7) and (8) are used to solve for $\Phi(x)$.

When $\omega_{ce} = 0$, it can be shown[8] that the classical Child-Langmuir result is obtained with

$$J_0(0) = J_{CL} = \frac{1}{9\pi}\left(\frac{2e}{m_e}\right)^{1/2}\frac{V^{3/2}}{D^2} \quad . \tag{9}$$

In this case, $n_e(D) = n_0 = J_{CL}/ev_0$. When $\omega_{ce} = \omega_{ce}^{crit}$, $v_x(D) = 0$ and the electrons reach the anode at grazing incidence. In this case, Eq. (8) can easily be solved for the eigenvalue $J_0(\omega_{ce}^{crit}) \equiv J_0^{crit}$, yeilding

$$J_0^{crit} = \frac{9J_{CL}}{4\pi} \approx 0.716 J_{CL} \quad . \tag{10}$$

It can also be shown[8] that $dJ_0/d\omega_{ce} \to -\infty$ as $\omega_{ce} \to \omega_{ce}^{crit}$. Thus, the electron emission abruptly shuts off at $\omega_{ce} = \omega_{ce}^{crit}$. The solutions of J_0/J_{CL} and $x^{(1)}/D$ (for $x^{(1)} < D$) as functions of $\omega_{ce}/\omega_{ce}^{crit}$ are shown in Fig. 2. Values of the various parameters are also shown in Table 1 for the special cases $\omega_{ce} = 0$, $\omega_{ce}^{(1)}$, and ω_{ce}^{crit}.

III. MINIMUM DIODE LENGTH FOR I_{crit}

If laminar flow is assumed in the diode gap, then the above analysis can be used to calculate the diode current $I_d(z)$ as a function distance along the cathode. $I_d(z)$ increases from zero at $z = 0$ to the total diode current I_{d0} at $z = L$ as shown in Fig. 1. The local self-magnetic field is given by $B_0(z) = 2I_d(z)/cR_c$ and determines the local current density from $J_0(\omega_{ce})$ as given in Fig. 2. Here, this local current density is denoted by $J_0(z)$ and $dI_d(z)/dz \equiv 2\pi R_c J_0(z)$. This diode configuration is illustrated in Fig. 1a for $I_{d0} << I_{crit}$, in Fig. 1b for $I_{d0} < I_{crit}$, and in Fig. 1c for $I_{d0} = I_{crit}$. The minimum diode length needed to obtain critical current then is determined by

$$I_d(L_{crit}) \equiv 2\pi R_c \int_o^{L_{crit}} J_0(z)dz = I_{crit} \quad , \tag{11}$$

where I_{crit} is defined in Eq. (2). A zero order estimate for L_{crit} can be obtained by assuming that $J_0(z)$ is uniform in z and equal to J_{CL}, yielding $L^0_{crit} = 9D/2\beta^2$, where, $\beta = v_0/c$. Results from integrating Eq. (11) are plotted in Fig. 3 and show that $L_{crit} \sim 1.07 L^0_{crit}$. This provides both scaling for and a reasonable estimate of the minimum diode length required to reach critical current.

If $\Delta(z)$ is the axial distance that an electron emitted at z is deflected by the magnetic field while crossing the gap (see Fig. 3),

$$\Delta(z) \equiv \int_0^T v_z dt = \int_0^D \frac{v_z(x)}{v_x(x)} dx \quad , \tag{12}$$

where T is the gap crossing time and the position z along the cathode is associated with the local value of $B_0(z)$ [or equivalently $\omega_{ce}(z)$]. Again, it is convenient to rewrite this integral in terms of S using

$$dx = \pm\left(\frac{32\pi e J_0}{m_e}\sqrt{S} - 4\omega_{ce}^2 S\right)^{-1/2} dS \quad ,$$

from Eq. (6), $v_x = S^{1/2}$ from the definition of S, and $v_z(x) = -\omega_{ce}x$ from Eq. (1) where Eqs. (7) and (8) are needed to write x in terms of S. When $x^{(1)} \geq D$, only Eq. (7) is needed. When $x^{(1)} < D$, the integral must be separated into two parts, using Eq. (7) for x in the interval $0 \leq x \leq x^{(1)}$ and using Eq. (8) in the interval $x^{(1)} \leq x \leq D$. $\Delta(z)$ is plotted in Fig. 3.

The analysis presented in this section remains valid until orbit crossing occurs. The orbit of an electron emitted from the cathode at $z + \delta$ crosses the orbit of an electron emitted at z if $z - \Delta(z) > (z + \delta) - \Delta(z + \delta)$. By Taylor expanding $\Delta(z + \delta)$ for $\delta << z$, it can be shown that the condition for orbit crossing becomes $d\Delta/dz > 1$. In order to evaluate this derivative, expressions for dJ_0/dz and $d\omega_{ce}/dz$ are needed. First, note that $dJ_0/dz = (dJ_0/d\omega_{ce})d\omega_{ce}/dz$ where the solution for $J_0(\omega_{ce})$ is shown in Fig. 2 and its differentiation with respect to ω_{ce} is straight forward. Then note that $d\omega_{ce}/dz =$

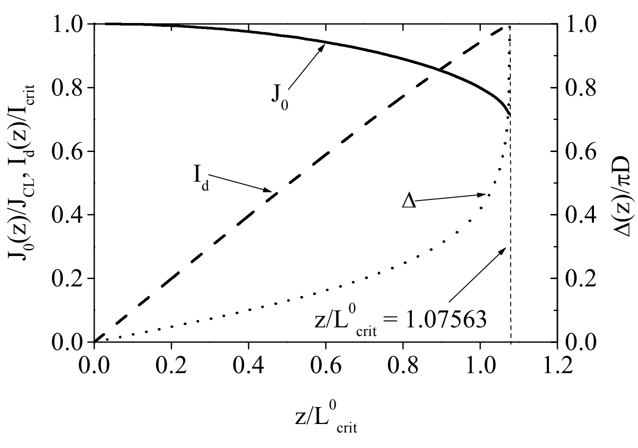

Figure 3. Plots of J_0/J_{CL} (solid), I_d/I_{crit} (dash), and $\Delta/\pi D$ (dot) as functions of z/L^0_{crit}.

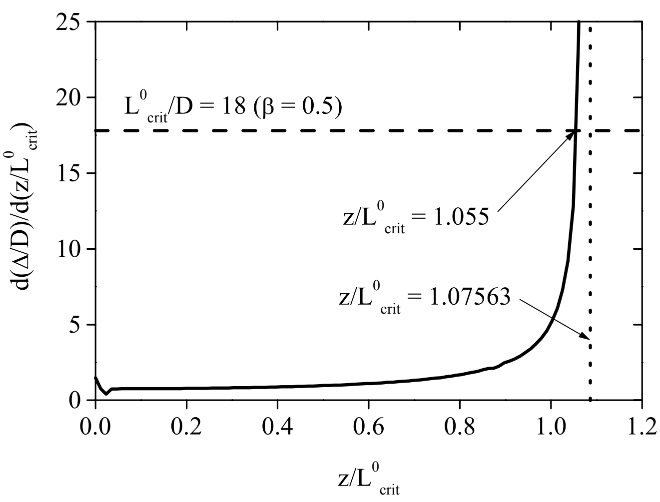

Figure 4. Plot of $d(\Delta/D)/d(z/L^0_{crit})$ as a function of z/L^0_{crit}.

Table 1. Special values for various parameters.

	$\omega_{ce} = 0$	$\omega_{ce} = \omega_{ce}^{(1)}$	$\omega_{ce} = \omega_{ce}^{crit}$
$\omega_{ce}/\omega_{ce}^{crit}$	0	$\pi/(\pi^2+4)^{1/2} \sim 0.84$	1
J_0/J_{CL}	1	$9\pi^2/2(\pi^2+4)^{3/2} \sim 0.86$	$9/4\pi \sim 0.72$
$x^{(1)}/D$	$+\infty$	1	0.5
$n_e(D)/n_0$	1	$9\pi^2/4(\pi^2+4) \sim 1.6$	$+\infty$
$v_x(D)/v_0$	1	$2/(\pi^2+4)^{1/2} \sim 0.54$	0
$v_z(D)/v_0$	0	$-\pi/(\pi^2+4)^{1/2} \sim -0.84$	-1
$\Delta(D)/D$	0	$(\pi^2-4)/2\pi \sim 0.93$	π

$(2e/m_ec^2R_c)dI_d/dz = 4\pi eJ_0(z)/m_ec^2$ where $I_d(z)$ and $J_0(z)$ are shown in Fig. 3. A plot of $d\Delta(z)/dz$ is shown in Fig. 4 where $\Delta(z)$ has been normalized to D and z has been normalized to L^0_{crit}. With this normalization, the condition for orbit crossing becomes

$$\frac{d(\Delta(z)/D)}{d(z/L^0_{crit})} > \frac{L^0_{crit}}{D} = \frac{9}{2\beta^2} \quad . \tag{13}$$

For this nonrelativistic analysis (i.e., $\beta << 1$), Eq. (13) and the results presented in Fig. 4 show that orbit crossing only occurs very close to L_{crit}. For example, orbit crossing first occurs at $z \sim 1.05L^0_{crit}$ for $\beta = 0.5$ (see dashed line in Fig. 4) and rapidly approaches $L_{crit} \sim 1.07L^0_{crit}$ (see dotted line in Fig. 4) as β decreases. Thus, this analysis is valid over nearly the full diode length.

As discussed in Sec. I, the minimum diode length required to reach critical current when ions are present decreases to $L^{BP}_{crit} = L_{crit}/1.86 = 2.6D/\beta^2$. This assumes that the ion current is much less than the electron current and that $D << R$, because it has been shown recently that the ion enhancement factor of 1.86 increases with aspect ratio R_c/R_a for cylindrical diodes with the cathode radius R_c larger than the anode radius R_a[9].

IV. SUMMARY

The emitted current density is found to only decrease modestly with magnetic field until the self-magnetic field associated with the critical current is reached where the emission shuts off abruptly. The 1-D analysis remains valid until orbit crossing occurs which has been shown to only occur very close to the full diode length when the current approaches the critical current. Thus, the estimate obtained for the diode length required to reach critical current is accurate. It is also predicted that, when ion current flows, the minimum diode length required to obtain critical current decreases by a factor of 1.86. Extending these results to the relativistic regime and including ions in the analysis will require a numerical solution of Poisson's equation and is the subject for future work.

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